Pre Calc Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

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 WS Assessment

Target 5

Rational function

* Simplify Expression
* Rational Functions / Asymptotes
* Graphing Rational Functions
* Inequalities: Rational and Radicals

HW 5 Rational Functions [www.deltamath.com](http://www.deltamath.com)

Simplify the following expression

4x2 (x – 2)3 + 2x(x – 2)4 $\frac{6(x^{2}+3)^{2}-6x\left(x^{2}+3\right)^{2}(4x)}{(x^{2}+3)^{4}}$

$\frac{x^{3}-8}{x^{2}-4}$ $\frac{x^{2}-2x-8}{x-4}$

$\frac{9-x^{2}}{9x^{2}(x-3)}$ $\frac{\frac{1}{x^{2}}-\frac{1}{9}}{x-3}$

$\frac{5}{3}x^{\frac{2}{3}}-\frac{10}{3}x^{\frac{-1}{3}}$ $x^{2}(\frac{1}{2})(x-2)^{\frac{-1}{2}}+2x\sqrt{x-2}$

$$\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

If  f(x)  is a rational function given by $f\left(x\right)=\frac{P(x)}{Q(x)}$where P(x) and Q(x) are polynomials, we can use the following information to sketch the graph of *f*

*I) Asymptotes* The value of the coordinates of points on the graph of *f* get arbitrarily large (in absolute value) as the graph approaches an asymptote but it will never crosses the asymptote.

**A. Vertical Asymptotes** To find the vertical asymptotes, we can first cancel any common factors in P(x) and Q(x) then take the vertical lines corresponding to **the zeros of the denominator**:

**B. Horizontal Asymptotes**

We can find the horizontal asymptotes by investigating the behavior of *f (x)* as x gets arbitrarily large (with either a plus sign or a minus sign):

1. If deg (P(x)) < deg Q(x)), then the **line** **y = 0 (x-axis) is the horizontal asymptote** for the graph of *f*.

2. If deg (P(x)) = deg Q(x)), and a and b are the coefficients of the highest powers of x appearing in P(x) and Q(x), respectively, then the **line y = a/b is the horizontal asymptote** for the graph of *f*.

3. If deg (P(x)) > deg Q(x)), then there is **no horizontal asymptote** for the graph of *f*.

**C. Slanted Asymptotes (in place of Horizontal Asymptotes – case #3)**

If deg (P(x)) = deg Q(x)) + 1, then the graph of *f* has a slanted asymptote; and we can find the slanted asymptote by dividing P(x) by Q(x): If P(x)/Q(x) = mx + b + Remainder where deg(Remainder) < deg(Q(x)) then the **line y = mx + b is the slanted asymptote**.

*II) The Intercepts* You know how to find these

*III) Sign Chart* See how it works in the example below

Example: Given $f\left(x\right)=\frac{(x+1)\left(x-1\right)^{2}(x-3)}{(x-2)}$ find the asymptotes and intercepts, then use this information and a sign chart to sketch the graph. (No calculator yet)

Solution Asymptote → Vertical \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Intercepts: y-intercept = ? (where x = 0) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

x-intercept = ? (where y = 0) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Sign Chart

|  |
| --- |
| **X = -1 1 2 3** |
| 0 |  |  |  |
|  | 0 |  |  |
|  |  | 0 |  |
| P(x) |  |  | 0 |
| Q(x):  |  |  |  |  |
| f(x) |  |  |  |  |

Sketch of the graph

For each function below do the following:

Find the asymptotes; Find the intercepts; Make a sign chart;

Determine if the graph of *f* crosses its horizontal asymptote, and if the graph has symmetry around the origin or the y-axis. Then use this information to sketch the graph of *f*

1. $f\left(x\right)=\frac{x+2}{x-2}$ 2. $f\left(x\right)=\frac{x}{x^{2}-4}$

3. $f\left(x\right)=\frac{2x^{2}-2}{x^{2}-9}$ 4. $f\left(x\right)=\frac{x^{2}+3x+2}{x^{2}-x-2}$

5. $f\left(x\right)=\frac{x-4}{x^{2}+x-2}$ 6. $f\left(x\right)=\frac{x^{2}}{16-x^{2}}$

7. $f\left(x\right)=x-4+\frac{3}{x}$ 8. $f\left(x\right)=x+\frac{1}{x-2}$

Finding holes of the function. Consider this rational function $f\left(x\right)=\frac{x^{2}-49}{2x^{2}-14x}$ by simplify it we have

Find the hole of the following function, Write your answer as coordinate point.

$f\left(x\right)=\frac{x^{2}-5x-6}{x^{2}+x}$ $f\left(x\right)=\frac{3x+21}{x^{2}-2x-63}$

Sketch the graph of the following function: (Show all asymptotes, intercepts, sign chart and hole)

$$f\left(x\right)=\frac{x^{2}-x-6}{2x^{2}+11x+14}$$

$$f\left(x\right)=\frac{5x^{2}+5x-30}{3x^{2}+9x}$$

Solve the following inequalities and sketch graph (desmos) to verify

$\frac{x^{2}-2x-24}{x^{2}-8x-20}\geq 0$ $\frac{x^{2}-5x+4}{x^{2}-4}\leq 0$

$\frac{2x^{2}-x-10}{-x^{2}+x+12}>0$ $\frac{2x^{3}-6x}{(x^{2}+1)^{3}}<0$

Find all values of x for which

$x< \frac{16}{x}$ $\frac{x}{x-4}< \frac{x-5}{x+1}$ $\frac{x-8}{x}<3-x$

$\sqrt{x^{2}-2x-8}>x+2$$\sqrt{2-x}<x$

**Target 5 Assessment**

Sketch the graph of the following function: (Show all asymptotes, intercepts, sign chart and hole)

$$f\left(x\right)=\frac{x^{2}-3x}{(x^{2}-4)(x-3)}$$

Find all values of x for which

$x\geq \frac{x+6}{x+2}$ $\sqrt{2x-5}>3-4x$